The model of the individual consumer

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Outline

1. Consumer preferences
2. Consumer budget constraint
3. Consumer equlibrium
4. Changing the equlibrium of the consumer
5. Income and substitution effects
6. Other methods of comparative statics
7. Economics of exchange

Consumer preferences

Rational individ

- knows exactly his preferences;
- can compare sets of goods, using their preferences;
- always chooses the set of goods that best suits his preferences.

The main components of consumer behavior

- Consumer goal is to receive maximum satisfaction from consumption of certain good sets.
- Budget restriction – these are all circumstances that do not allow the consumer to get everything he wants. The most important of these are the prices of goods and services and consumer income.
- Consumer choice is to make and implement a decision on the volume and structure of the consumer set under these restrictions, which would maximize the satisfaction of needs.

Approaches to the analysis of consumer behavior

- cardinal or quantitative
- ordinary or ordinal

The essence of the cardinalist approach

- utility maybe to be measured quantitatively by help conditional units – «utils»;
- the consumer evaluates consumer property of each product in utilities and chooses goods with the largest number;
- the value of utility depends not only on the properties of the good, but also on its quantity, that is, determined functionally.

Total utility

- The total amount of satisfaction that the consumer receives from all the goods consumed is called total utility (TU).The dependence of total utility on the number of consumed goods is reflected by the function:

$$TU = f(X, Y, \ldots),$$

where X, Y, ... – the number of consumed goods.

Marginal utility

- Marginal utility (MU) is the additional utility derived from the consumption of an additional unit of good, or the increase in total utility when changing the amount of good per unit.

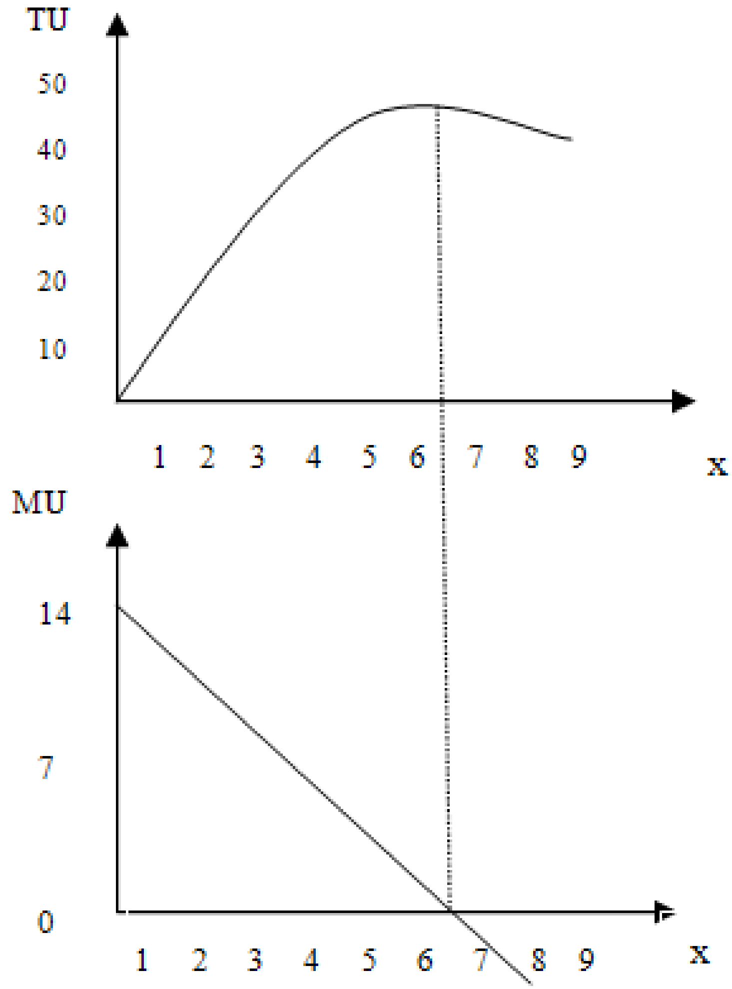
The law of diminishing marginal utility

- The amount of satisfaction from the consumption of each additional unit of goods of this type decreases until it reaches zero at the point of full satisfaction of needs.

Example 1

Amount of good X	Aggregate utility, utils	Marginal utility MU_X, utils
0	0	0
1	12	12
2	22	10
3	30	8
4	36	6
5	40	4
6	42	2
7	42	0
8	40	–2

Functions of total and marginal utility



Advantages of the cardinalist approach

- Providing a simple explanation of the motivation of consumer behavior
- Ability to analyze the choice of benefits – two, three and greater quantity of goods.

The lack of a cardinalist approach

- consumer is not able quantitatively to evaluate the difference in goods to determine, for example, in how many utils is loaf bread more useful than a package of milk.

The essence of the ordinaryist model

- Ordinalistic model is a system that determines which of the two sets of goods the consumer prefers.

Assumptions–axioms underlying the approach – 1

- Axiom of completeness: the rational consumer can always choose the best of any two sets, or say that they are equivalent to him:

$$(x, y): x \succ y \cup x \prec y \cup x \sim y$$

- Axiom of transitivity: the advantage among different sets is consistent for all sets of goods: $x \succ y, y \succ z \Rightarrow x \succ z$

Assumptions–axioms underlying the approach – 2

- Axiom of reflectivity: any set is at least not worse than itself:

$$\forall x \quad x \geq x$$

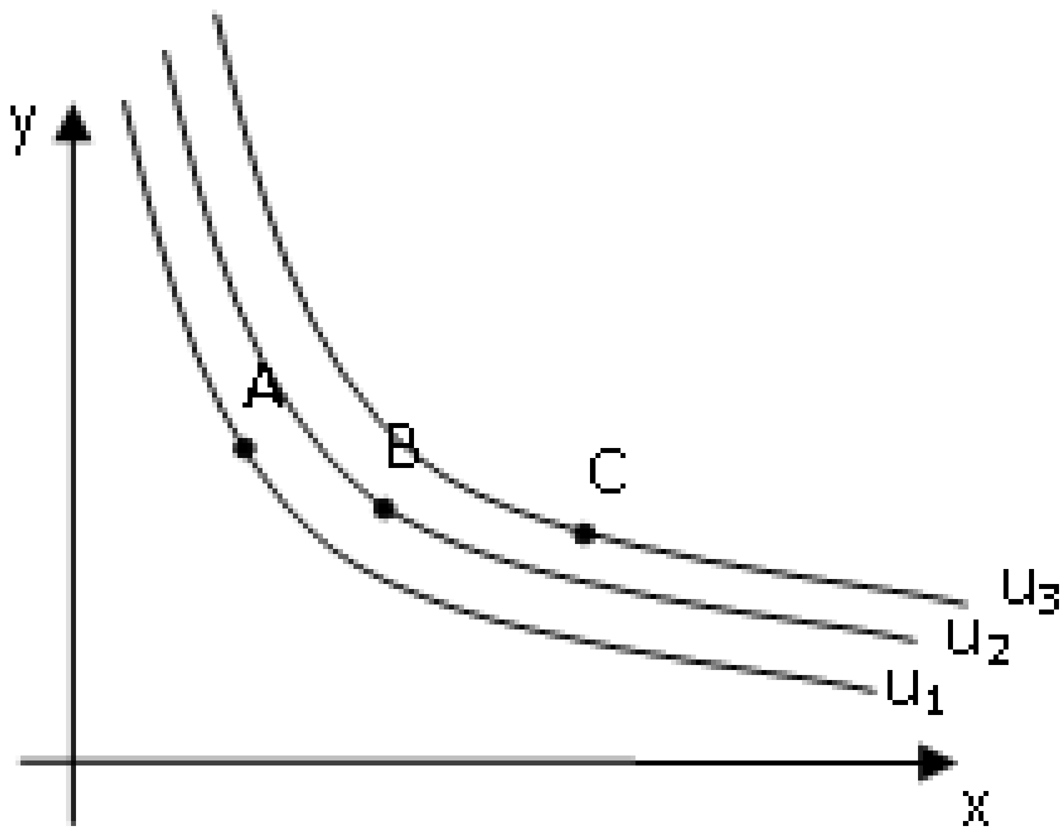
- Axiom about insatiability: the consumer is never satisfied, in the sense that a set with more items is better for him than a set with fewer.

- Axiom of independence: consumer satisfaction depends only on the amount of goods consumed by him, and does not depend on the amount of goods consumed by others.

Indifference curves and indifference curve maps

- The indifference curve is the line on which all sets of goods equivalent to the consumer are located.
- The set of indifference curves for a consumer is called the indifference curve map. Consumer indifference curves never intersect.

Map of indifference curves

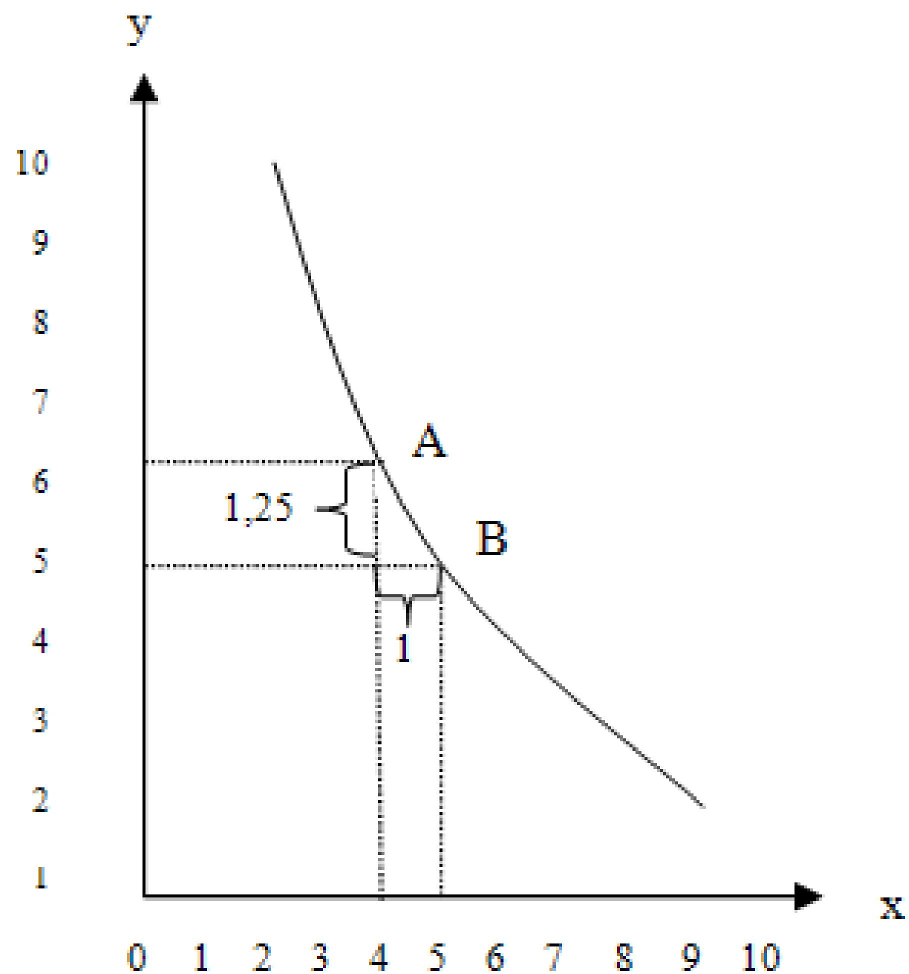


Marginal rate of substitution

- The amount of good y that the consumer is forced to give up in order to receive an additional unit of good x is called the marginal rate of substitution (MRS). It can be defined as the angular coefficient of the indifference curve at each point:

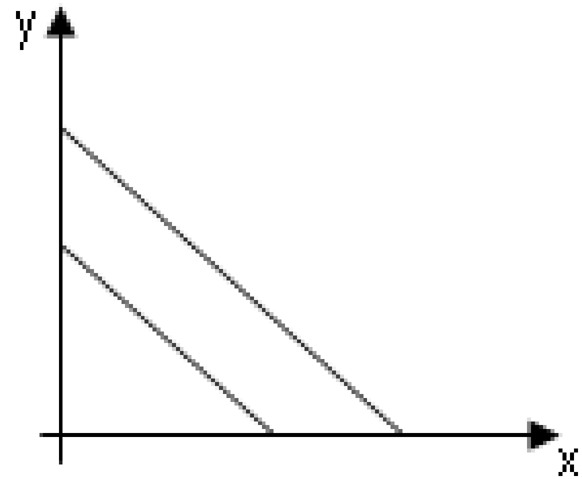
$$MRS_{xy} = \left|\frac{\Delta y}{\Delta x}\right|$$

Example 2



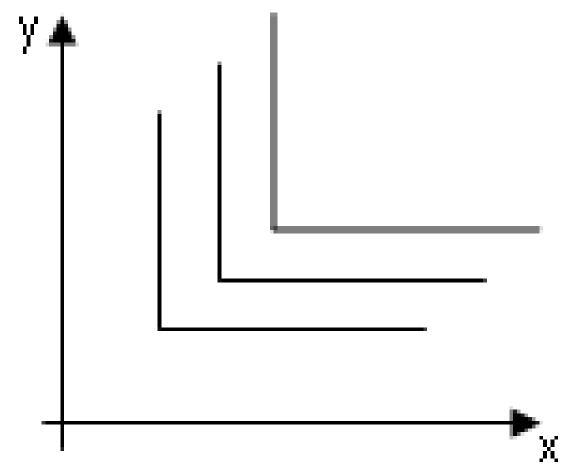
Special types of indifference curves – 1

- Perfect substitutes – goods that can be completely replaced by one another with a constant replacement factor.



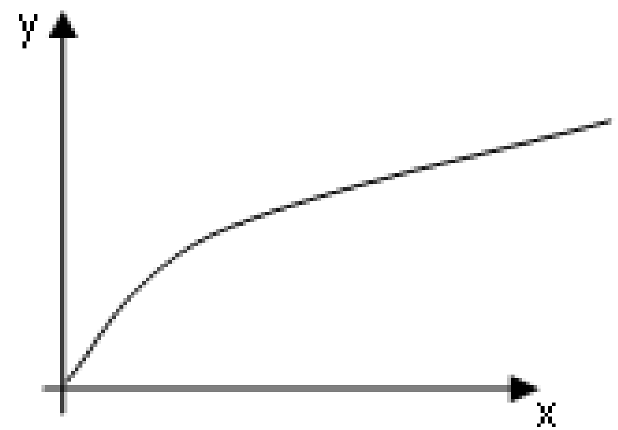
Special types of indifference curves – 2

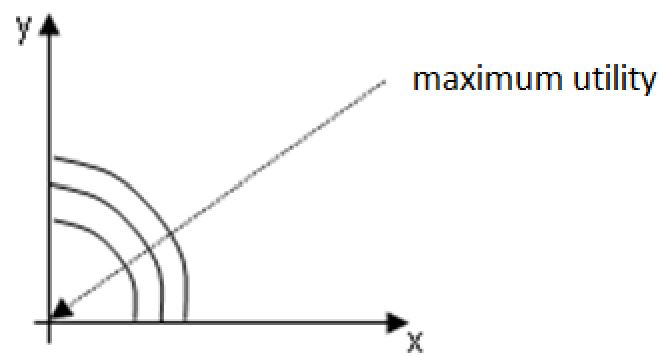
- Perfect complements are goods that can be consumed only in a fixed proportion.



Special types of indifference curves – 3

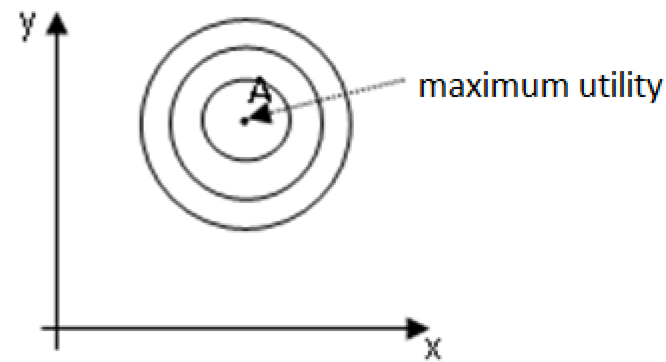
- Harmful goods are the group of goods whose consumption is harmful. The individual agrees to consume such goods only with some compensation





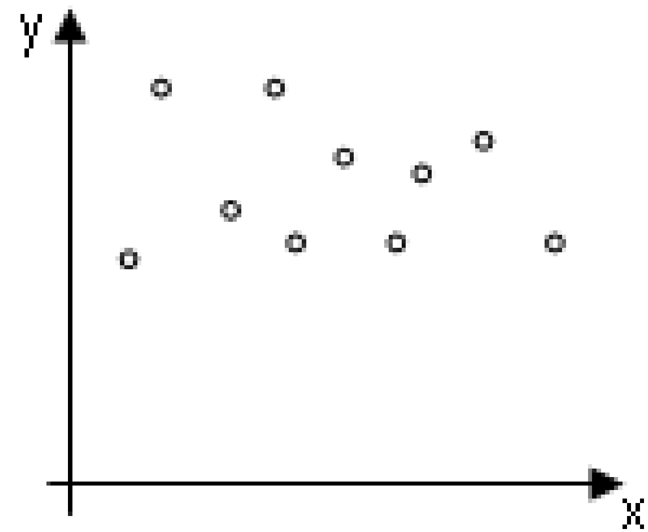
Special types indifference curves – 4

- Saturation goods – the individual knows which set of goods will satisfy him in the best way, the farther from this set the individual is, the less satisfaction he will receive.



Special types indifference curves – 5

- Discrete goods – goods that can be consumed in a certain amount.



Consumer budget constraint

Budget constraint

- x, y – goods, p_x, p_y – prices for them, b – the amount of money.
- The budget constraint is as follows:

$$p_x x + p_y y \leq b$$

Slope of budget constraint

Let

x, y – two goods,

p_x, p_y – prices for them,

b – amount of money,

then the budget constraint is:

$$p_x x + p_y y \le b$$

Slope of the budget constraint curve:

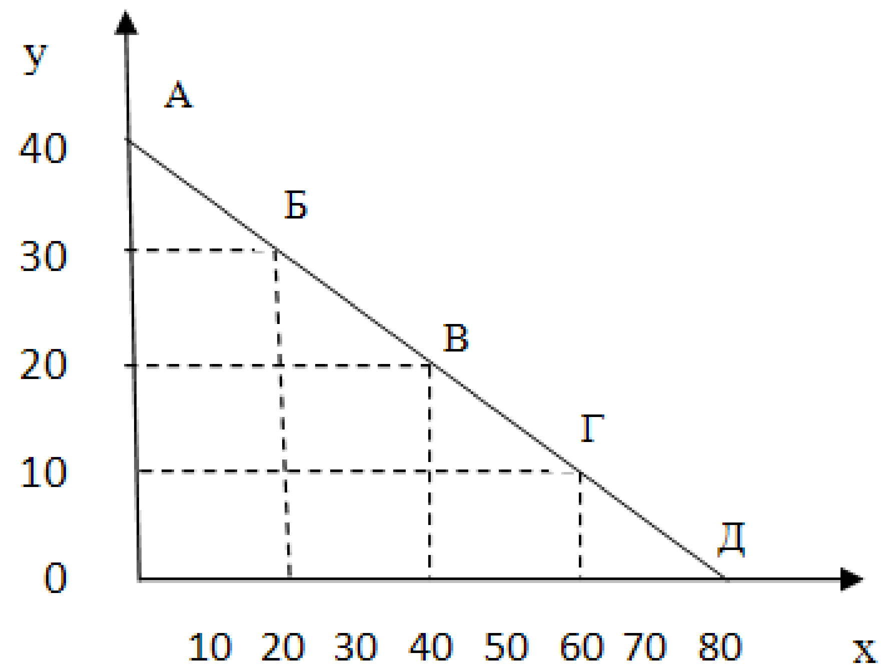
$$k = -p_x / p_y$$

Example 3

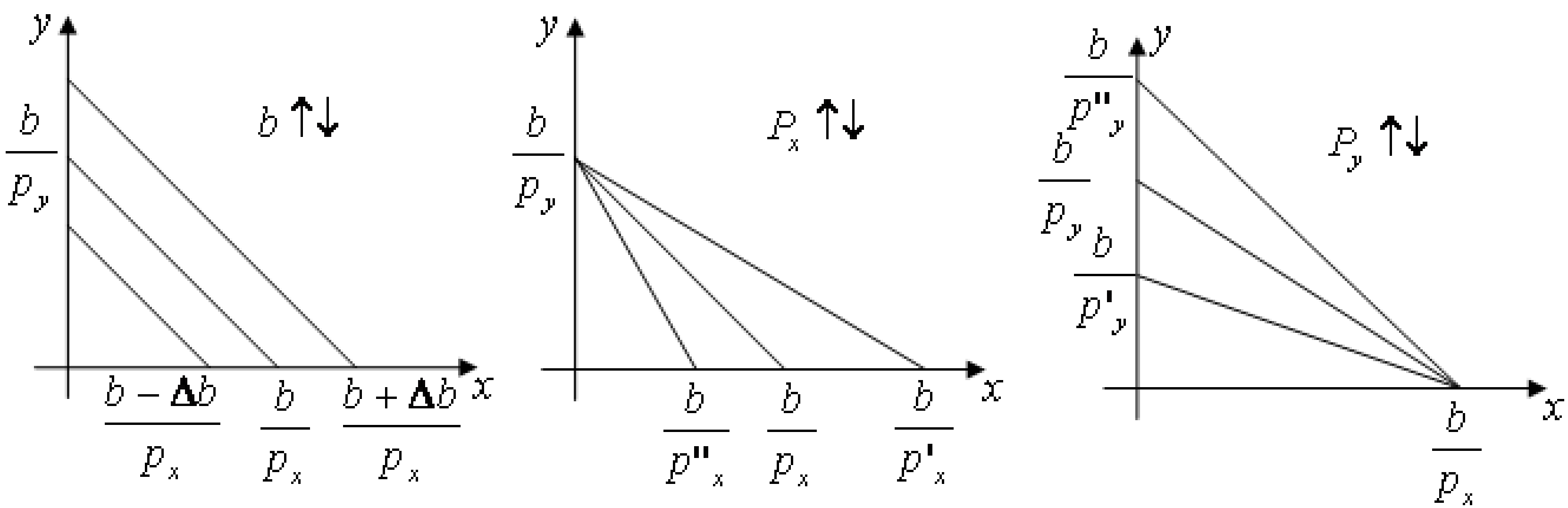
- The weekly income of the consumer is 80 UAH. and is entirely spent on the purchase of two goods, the prices of which are 1 USD for goods x, and 2 USD for good y. Available sets:

Sets	A	B	C	D	E
Freight x	0	20	40	60	80
Goods in	40	30	20	10	0

Consumer budget constraint



The impact of changes in commodity prices and money on budget constraints

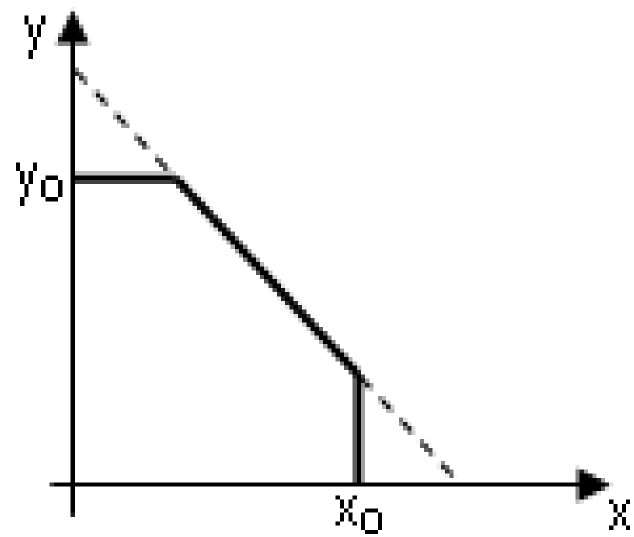


Type of budget constraint

- The form of budget constraint in addition to the prices of goods and revenues may be influenced by other factors, including various types of discounts, benefits or subsidies, time constraints on transactions.

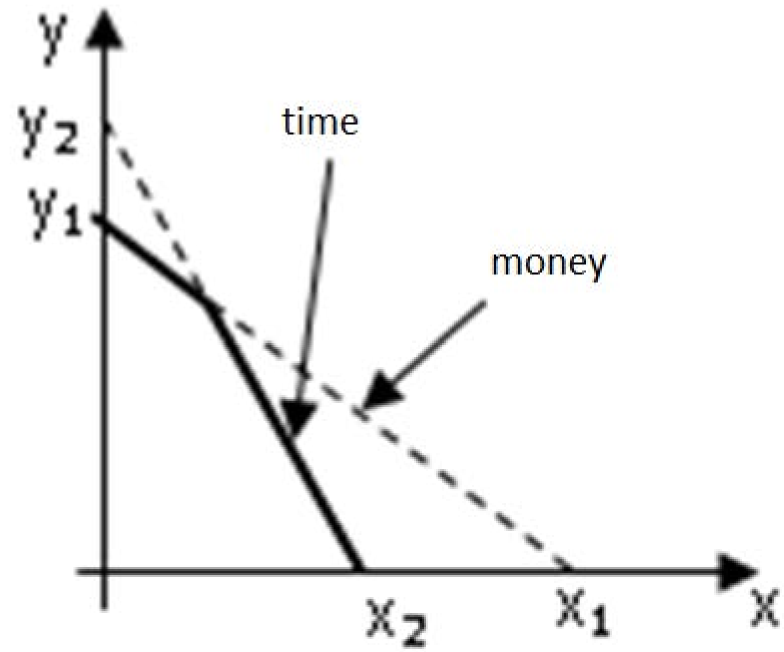
Forms of budget restrictions – 1

- Let the consumer not be able to buy the product x more than x_0 and a product y more than y_0.



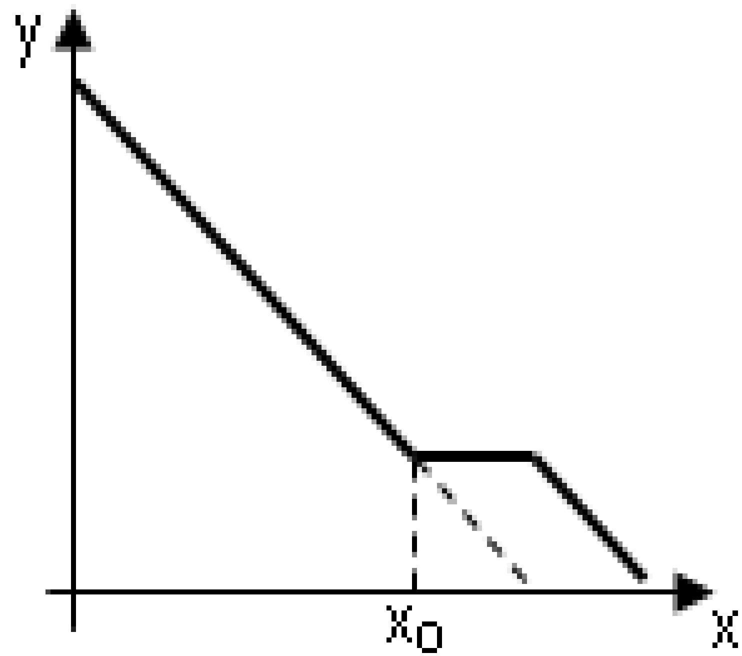
Forms of budget restrictions – 2

- Let the important factor influencing is time



Forms of budget restrictions – 3

- Government grants

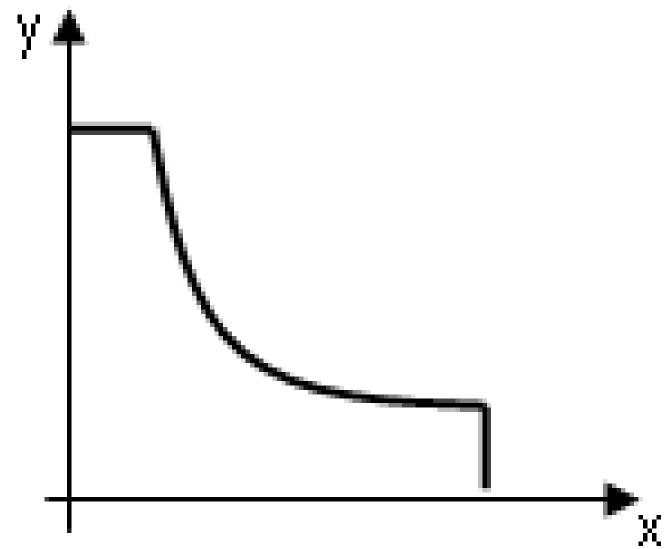


Forms of budget restrictions – 4

- Flexible system of discounts

$$x_1 < x_2 < \ldots < x_n$$

$$p_1 > p_2 > \ldots > p_n$$



Consumer equilibrium

Consumer equilibrium

- Equilibrium of the consumer is a situation in which the consumer has no incentive to change their behavior (purchase of another set).
- Equilibrium is the solution to the problem of maximizing utility with a given budget constraint:

$$\begin{cases} U(x_1, x_2, \dots, x_n) \to \max \\ (p, x) \le b \end{cases}$$

The optimal point meets 2 conditions:

- Located on a budget line
- Gives the consumer maximum satisfaction

Utility maximization rule

$$\frac{MU_x}{p_x} = \frac{MU_y}{p_y} = \ldots = \frac{MU_n}{p_n}$$

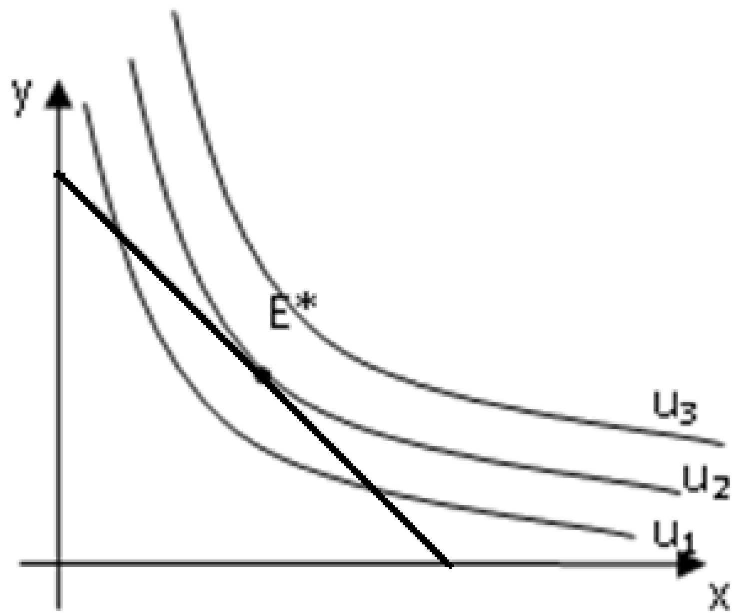
The rule of optimizing consumer choice

- The choice is optimal if, within the budget constraint, the ratio of marginal utility of any type of goods is equal to the ratio of their prices:

$$\begin{cases} \dfrac{MU_x}{p_x} = \dfrac{MU_y}{p_y} \\ p_x x + p_y y \le b \\ x, y \ge 0 \end{cases}$$

Optimal set

- Optimal set located in point touch the highest available to the consumer curve indifference to budget lines.



Consumer equilibrium: substitute goods

$$\begin{cases} u(x,y) = \max\left\{\dfrac{x}{k_x}; \dfrac{y}{k_y}\right\} \\ p_x x + p_y y = b \end{cases}$$

$$\frac{x}{y} = \frac{k_x}{k_y}$$

$$u = \max\left\{\frac{b}{p_x k_x}; \frac{b}{p_y k_y}\right\}$$

$$=> \begin{pmatrix} x^* = \dfrac{b}{p_x} \\ y^* = 0 \end{pmatrix}, u = \frac{b}{p_x k_x}$$

$$\begin{pmatrix} x^* = 0 \\ y^* = \dfrac{b}{p_y} \end{pmatrix}, u = \frac{b}{p_y k_y}$$

Consumer equilibrium: complementary products

$$\begin{cases} u = \min\{x_1; x_2\} \\ p_x x + p_y y = b \\ \dfrac{x}{y} = \dfrac{k_x}{k_y} \end{cases}$$

$$u^* = \frac{x^*}{k_x} = \frac{y^*}{k_y}$$

Change of consumer equilibrium

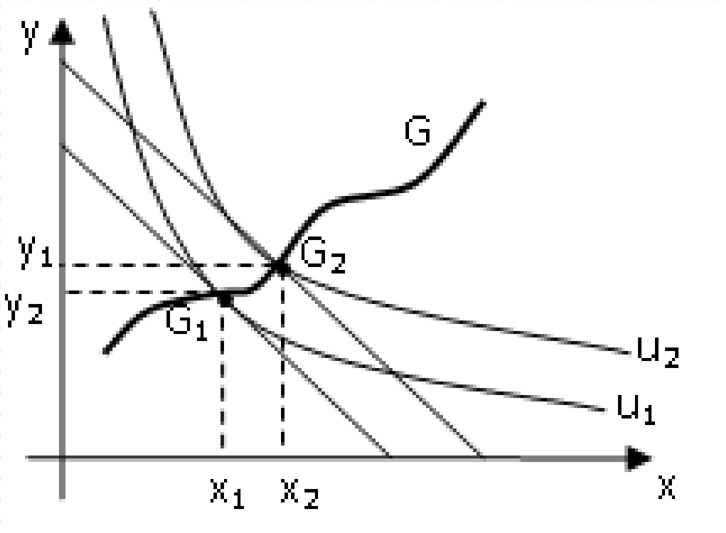
Comparative statics

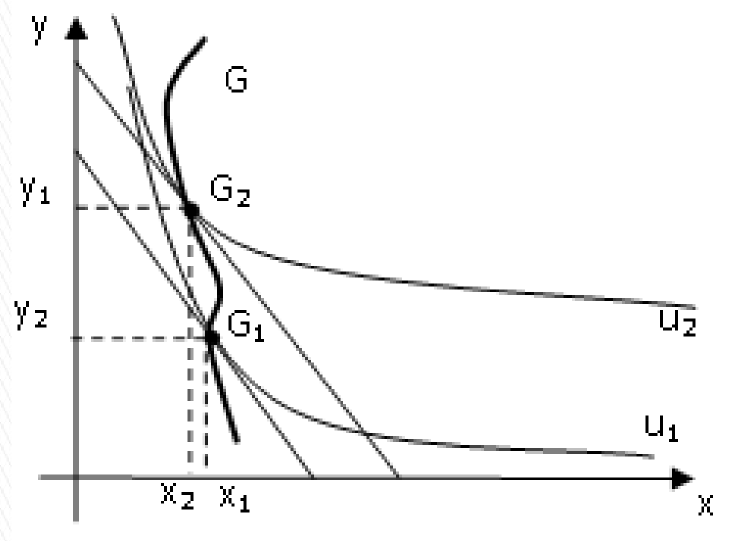
- The process of comparing two equilibria is called "comparative statics".
- The method of comparative statics is a procedure of comparing two equilibria without analyzing the dynamics of how the consumer moves from one state of equilibrium to another.
- Analysis of comparative statics allows you to easily convey how the consumer will behave in different situations and determine the curves of consumer demand.

Income – consumption curve – 1

- The income–consumption curve is the set of all optimal sets or combinations of goods with changing consumer income and constant price ratio.

Income-consumption curve- 2





If the income-consumption curve has a positive slope, then such goods are called normal.

If with the growth of income the consumption of one commodity increases and the other decreases, then the first commodity is called low-quality or low-quality commodity, and the second commodity is called high-quality.

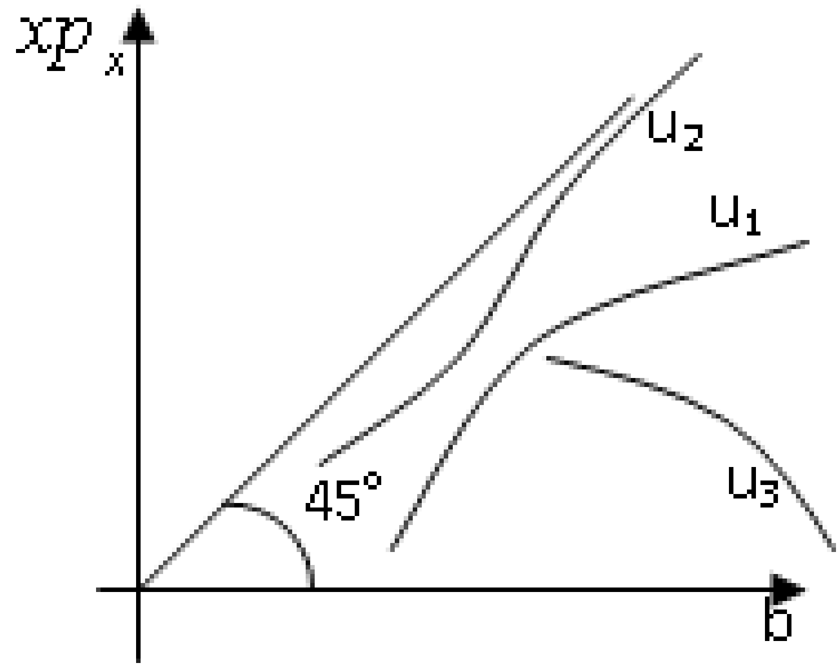
Engel's curve

- The Engel curve shows the relationship between the volume of consumption of goods and consumer income at constant prices and benefits.
- The Engel curve shows the differences between normal, low-quality and high-quality goods, analyzing the costs of aggregate groups of goods (food, non-food, services, etc.).

The slope of the Engel curve

On the slope of the curve Engel you can determine the quality of the goods

- u1 – normal goods
- u2- high quality product
- u3- low quality goods

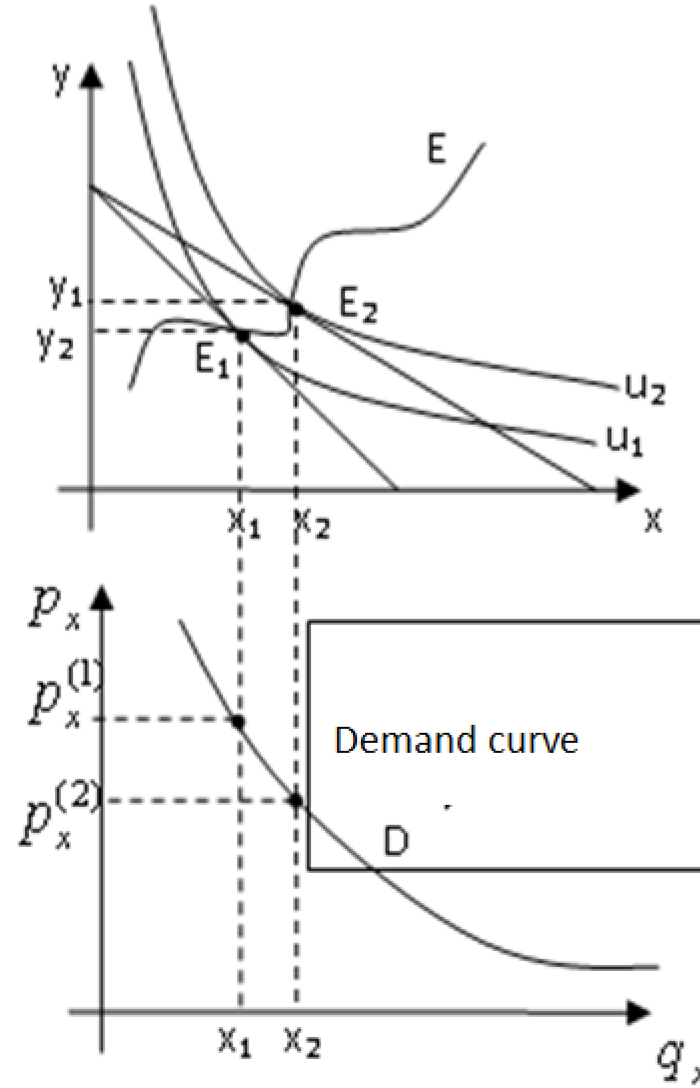


Optimal choice and price change

- Let's say, what by others equal conditions price goods *x* decreases.
- Connecting points balance smooth line, obsessed curve «price – consumption».

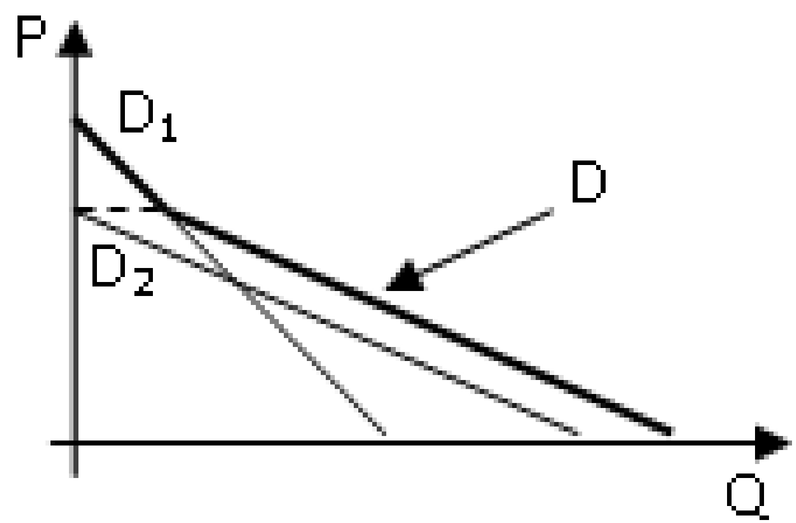
Individual demand curve

- The individual demand curve is a curve that shows how much economic good a consumer is willing to buy at different prices at a given time.
- It has a negative slope, indicating the desire of consumers to buy more goods at a lower price.



Market demand curve

Given the known curves of individual demand can be built ***curve of market (general) demand***:



Factors affecting market demand:

- Increase / decrease in consumer income;
- Change in prices of substitute goods in supplementary goods;
- Increasing or decreasing the number of buyers in the market;
- Price and deficit expectations;
- Changing consumer tastes and preferences;
- Psychological aspects.

Income and substitution effects

Income and substitution effects – 1

- **Income effect** – this is the effect that arises due to changes in real income.
- **Substitution effect** – this effect is only due to changes in relative prices compensating for changes in real income.

Approaches to determine the real income

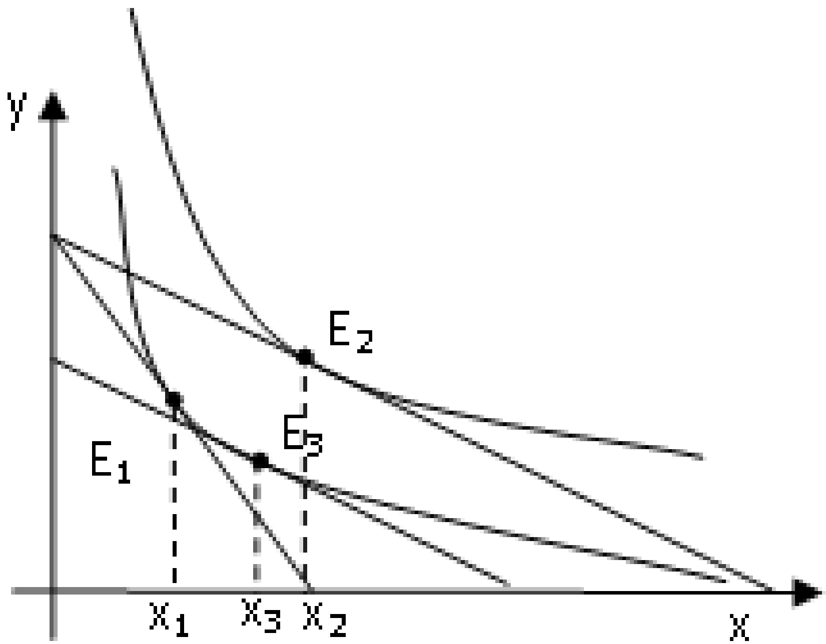
- by Hicks: different levels of cash income that provide the same level of satisfaction (allowing you to achieve the same indifference curve) represent the same level of real income.
- by Slutsky: only the level of cash income that is sufficient to purchase the same set provides a constant level of real income.

Income and substitution effects

Goods	Substitution effect	Income effect	Overall effect
Normal	<0	<0	<0
Low quality	<0	> 0	<0
Giffen goods	<0	> 0	> 0

Income effect and substitution effect by Hicks

- Let the price of the goods *x* decreases:



$$x_2 - x_1 = TE - total\ effect$$
$$x_3 - x_1 = SE - substitution\ effect$$
$$x_2 - x_3 = IE - income\ effect$$
$$TE = SE + IE$$

Algorithm for finding points E1, E2, E3

- find the point E1 (x1, y1):

$$\begin{cases} u(x,y) \to \max \\ p_x x + p_y y \le b \\ x, y \ge 0 \end{cases}$$

- calculate the point E2 (x2, y2):

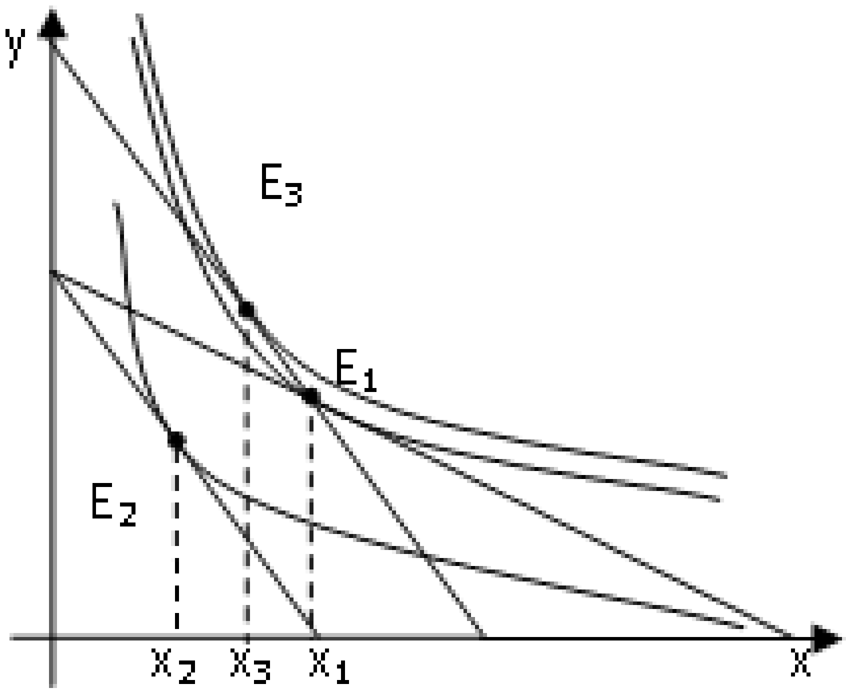
$$\begin{cases} u(x,y) \to \max \\ p_x' x + p_y y \le b \\ x, y \ge 0 \end{cases}$$

- find the point E3 (x3, y3):

$$\begin{cases} u(x,y) = u^* \\ p_x' x + p_y y \le b' = b + \Delta b \\ x, y \ge 0 \end{cases}$$

where is the level of satisfaction at point E1 $u^*(x_1, y_1) = u(x_1, y_1)$

Income effect and substitution effect by Slutsky



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Algorithm for finding points E1, E2, E3

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$$\begin{cases} u(x,y) \to \max \\ p_x' x + p_y y \le b \\ x, y \ge 0 \end{cases}$$

- find the point E3 (x3, y3):

$$\begin{cases} u(x,y) \to \max \\ p_x' x + p_y y \le b' \\ x, y \ge 0 \end{cases}$$

where

$$b' = p_x' x_1^* + p_y y_1^*$$

Example

The consumer utility function is as follows:

$$U(x_1, x_2) = x_1 x_2^{\,2}$$

The price of the goods x_1 is 3 USD, x_2 – 2 USD. The consumer budget is USD100. The price of the good x_1 decreases by USD1. What will be the income and substitution effects of by Hicks and Slutsky? What will be the overall effect?

Solution – 1

- Point E1:

$$\begin{cases} x_1 x_2{}^2 \to \max \\ 3x_1 + 2x_2 = 100 \end{cases} \Rightarrow x_1 = \frac{100}{9}, \quad x_2 = \frac{100}{3}$$

$$E_1 = (\frac{100}{9}; \frac{100}{3})$$

- Point E2:

$$\begin{cases} x_1 x_2{}^2 \to \max \\ 2x_1 + 2x_2 = 100 \end{cases} \Rightarrow x_1 = \frac{50}{3}, \quad x_2 = \frac{100}{3}$$

$$E_2 = (\frac{50}{3}; \frac{100}{3})$$

Overall effect:

$$E_2 - E_1 = \frac{50}{3} - \frac{100}{9} = \frac{50}{9}$$

Solution -2

- Point E3 by Hicks

$$u^* = \frac{100}{9} * \left(\frac{100}{3}\right)^2 = \frac{1000000}{81}$$

$$\begin{cases} x_1 x_2^{\ 2} = \dfrac{1000000}{81} \\ 2x_1 + 2x_2 = 100 \end{cases} \Rightarrow$$

$$x_1 = \left(\frac{500}{9}\right)^{\frac{2}{3}} = 14,56; \quad x_2 = 35,44$$

$$E_3 = (14,56; 35,44)$$

Substitution effect = 14.56-100 / 9 = 3.44
Income effect = 50 / 3-14.56 = 2.11

- Point E3 by Slutsky

$$b' = 2 * \frac{100}{9} + 2 * \frac{100}{3} = \frac{800}{9}$$

$$\begin{cases} x_1 x_2^{\ 2} \to \max \\ 2x_1 + 2x_2 = \dfrac{800}{9} \end{cases} \Rightarrow$$

$$x_1 = \frac{400}{27}, \quad x_2 = \frac{800}{27}$$

$$E_3 = \left(\frac{400}{27}; \frac{800}{27}\right)$$

Effect substitution = 400 / 27-100 / 9 = 3.70
Effect income = 50 / 3-400 / 27 = 1.85

Other methods of comparative statics

Elasticity – 1

- Elasticity – the ratio of the percentage change from one indicator to another:

$$E = \frac{\Delta x\%}{\Delta y\%}$$

- Elasticity of demand by price – an indicator of the percentage change in demand when the price changes by 1% (for normal goods <0):

$$E_p^D = \frac{\Delta Q / Q}{\Delta p / p}$$

- Arc elasticity:

$$E_p^D = \frac{\Delta Q}{(Q_1 + Q_2)/2} \Bigg/ \frac{\Delta p}{(p_1 + p_2)/2}$$

Elasticity – 2

- Elasticity of demand on income – an indicator of the percentage change in demand with a change in income by 1%:

$$E_b^D = \frac{\Delta Q}{Q} \Big/ \frac{\Delta b}{b}$$

- Cross-elasticity of demand – the elasticity of demand for one good, relative to prices for another good:

$$E_{yx}^D = \frac{\Delta Q_x}{Q_x} \Big/ \frac{\Delta p_y}{p_y}$$

Price index – 1

- Cost or income index – the ratio of cash income or loss of the current period to cash income or expenses of the base period:

$$I_E = \frac{\sum_{i=1}^{n} x_i^t p_i^t}{\sum_{i=1}^{n} x_i^0 p_i^0}$$

Laspeyres and Paashe indices

- the ratio of the value of the goods of the base period in the prices of the current period to the value of the goods of the base period in the prices of the base period

$$I_L = \frac{\sum_{i=1}^{n} x_i^0 p_i^t}{\sum_{i=1}^{n} x_i^0 p_i^0}$$

Laspeyres Index

- the ratio of the value of goods of the current period in the prices of the current period to the value of goods of the current period in the prices of the base period

$$I_P = \frac{\sum_{i=1}^{n} x_i^t p_i^t}{\sum_{i=1}^{n} x_i^t p_i^0}$$

Paashe Index

Fisher's index

- geometric mean indices Laspeyres and Paashe

$$I_F = \sqrt{I_L \cdot I_P}$$

Economics of exchange

Distribution of goods

- The distribution of goods is effective when a given volume of products produced over a period of time is distributed among consumers in such a way that it becomes impossible to improve the situation of one person without harming another.

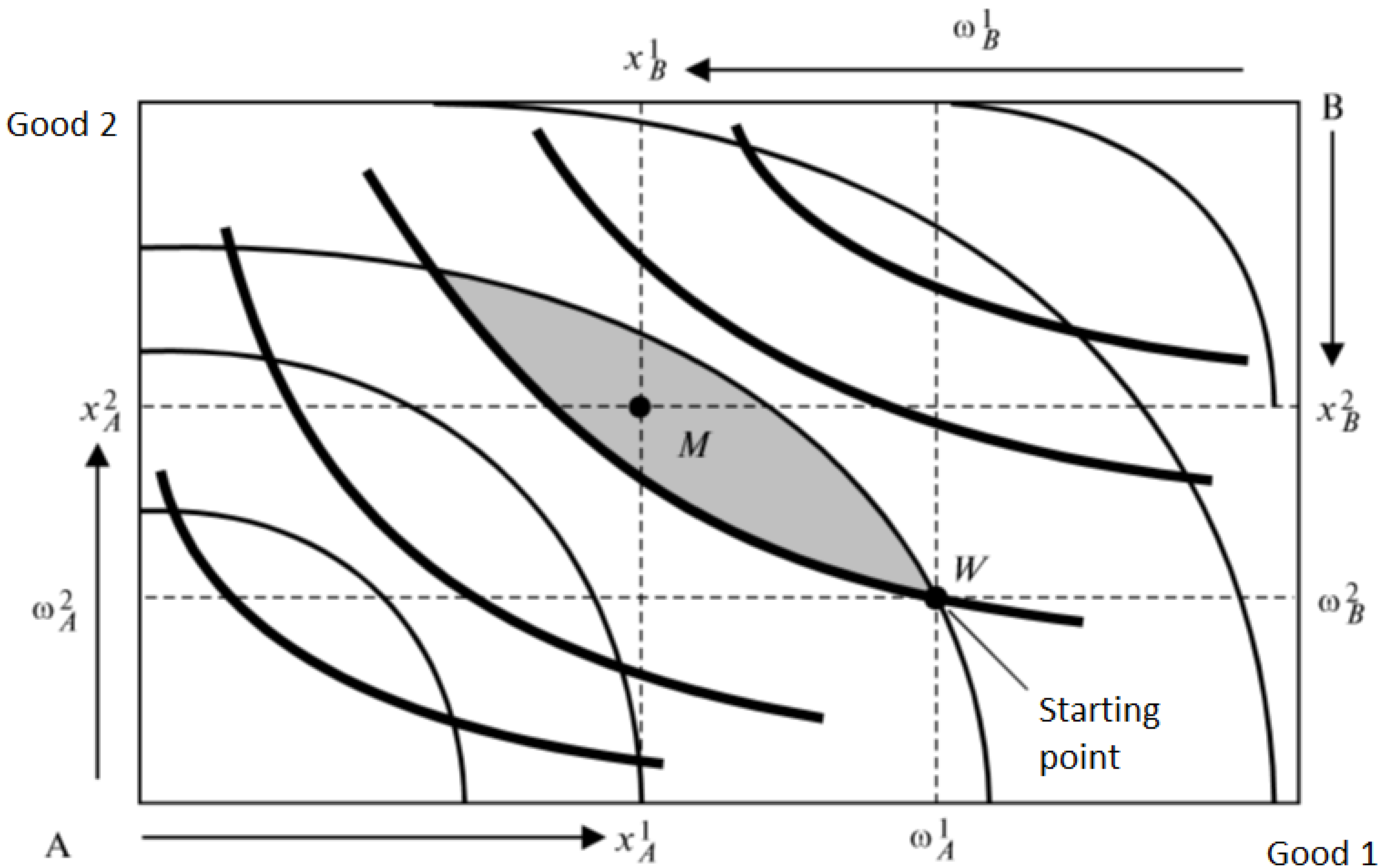
Efficiency in exchange

- Let the goods F and C be distributed among consumers A and B. The total number of goods is fixed.
- Effective options for the distribution of products F and C between consumers A and B are presented in the Edgeworth box.
- At the points of effective distribution:

$$P_F / P_C = MRS_{FC}^{A} = MRS_{FC}^{B}$$

- It is impossible to make a net profit by exchanging products.

Efficiency in exchange: chart Edgeworth



Example

Two households A and B consume two goods x_1 and x_2 , having such utility functions $u_A = x_1^A x_2^A$ and $u_B = x_1^B x_2^B$. First A and B have sets $(e_1^A, e_2^A) = (2,6),\ (e_1^B, e_2^B) = (10,4).$

- Determine area, which is the best and the worst for Pareto. Determine conditions for the Pareto optimum.
- Build contract curve for this economy.
- Which distributions of goods are possible, coming out with initial situations?

Solution – 1

Optimum condition for Pareto:

$$MRS_A = MRS_B$$

$$\frac{x_2^A}{x_1^A} = \frac{x_2^B}{x_1^B}$$

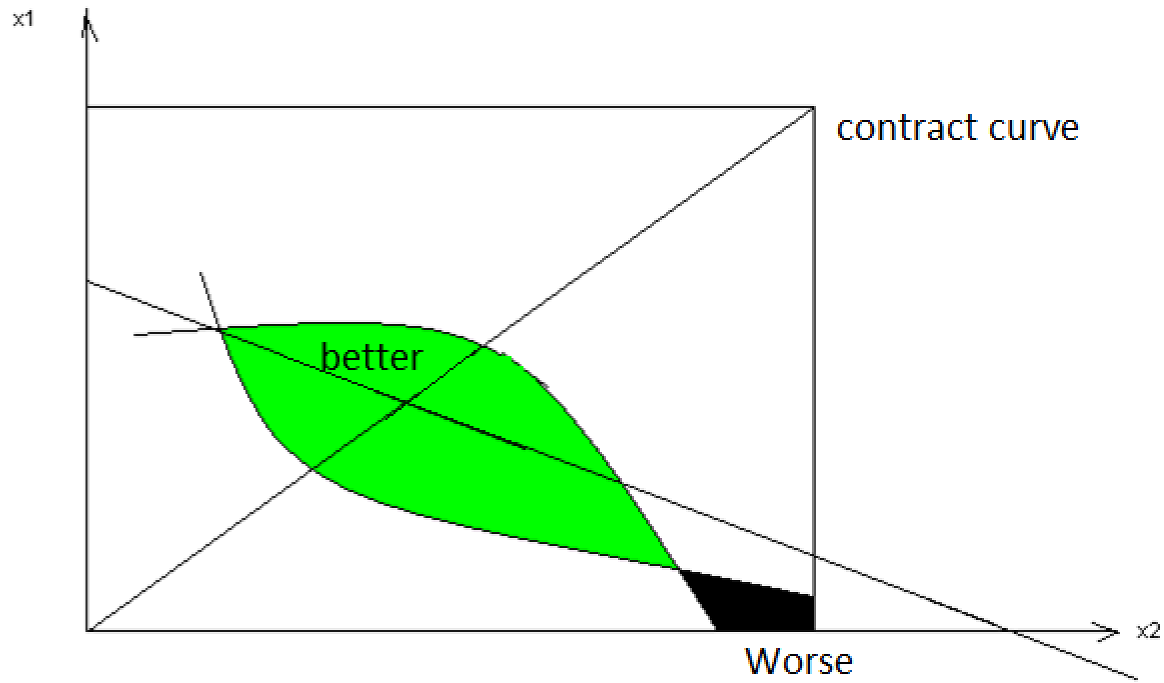
$$x_2^B = 10 - x_2^A, x_1^B = 12 - x_1^A$$

$$\frac{x_2^A}{x_1^A} = \frac{10 - x_2^A}{12 - x_1^A}$$

$$12x_2^A - x_1^A x_2^A = 10x_1^A - x_1^A x_2^A$$

$$x_2^A = \frac{5}{6}x_1^A$$

Solution – 2



Thank you!